ABSTRACT
We discuss cognitive aspects of modeling and simulation in an efficacy study of computational pedagogical content knowledge professional development of K-12 STEM teachers. Evidence includes data from a wide range of educational settings over the past ten years. We present a computational model of the mind based on an iterative cycle of deductive and inductive cognitive processes. The model is aligned with empirical research from cognitive psychology and neuroscience and it opens door to a whole series of future studies on computational thinking.

General Terms
Computational Theory of Mind, K-12 Teaching and Learning

Keywords
Deductive and Inductive, Cognitive Processes, Memory Retrieval

1. INTRODUCTION
Educators structure training and curriculum based on learning theories of how the human mind works. Recent findings from empirical research by cognitive psychologists and neuroscientists have created a critical mass to change the way we prepare teachers and support their classroom instruction. This is an opportune time for computer science educators to ground in cognitive theories the well-known concepts and processes in computational science.

Make it Stick, an ostensibly groundbreaking book published recently and coauthored by several prominent cognitive scientists has turned conventional ideas of learning upside down (Brown et al. 2014). The book offers many sound practices to help students easily retrieve content they learned in class, retain it, and apply it in different contexts to solve problems. New research suggests that repeated, delayed and interleaved retrievals make new concepts stick in memory longer if the process is effortful (pp. 47). Learning is mediated by memory, because human brain attempts to interpret new concepts in terms of previously registered knowledge and facts. However, some degree of forgetting is also good for learning because it forces the learner to use effort to cognitively engage oneself to recall or reconstruct newly acquired concepts through different neural pathways or links that exist and are retrievable.

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According to neuroscience, information is stored into the memory in the form of a specific pattern of neurons placed on a pathway and fired together (Restak 2001, Brown et al. 2014). The number and strength of such pathways improve the storage and retrieval of information. A memory or a newly learned concept can be a combination of previously formed memories, each of which might also involve a vast network of concepts and details mapped onto the brain’s neural network in a hierarchical way shown in Fig. 1.

The key to storing a concept more permanently into the memory is to link it to previously stored basic and retrievable concepts. And, the more links to associated concepts, the higher the chances of recalling this concept when needed later. Spaced-out cognitive retrieval practices attempted at different times, various settings and contexts is good because every time the recall is attempted it establishes more links that will help the remembering and learning. Exposure to new concepts through links to multiple views from different fields of study is, therefore, an effective retrieval strategy recommended by cognitive psychologists (Brown et al. 2014). This is called interleaved retrieval practice and it now forms a cognitive foundation for the computational pedagogical content knowledge (CPACK) framework that we developed for teacher professional development (Yaşar et al. 2015). In the following Sections (2.1 - 2.5) we describe theoretical foundation of CPACK followed by its implementation and impact on teaching and learning (Sec. 3) in secondary school classrooms.

Figure. 1: Distributive and associative aspects of information storage and processing (Yaşar 2015).
2. THEORETICAL FOUNDATION

2.1 Interdisciplinary Education

Interleaving retrieval practices by weaving together multidisciplinary features around a common topic (i.e., interdisciplinary education) has great advantages for gaining deep and lasting knowledge but it is not easy for several reasons. It would require a more cognitive effort than usual and as such, it would slow down the process of learning. In college, it would delay graduation and in public schools’ packed schedules it would risk compliance with local and state-mandated curriculum. Technology can be used to speed up this interdisciplinary learning but it needs training of teachers to teach content in pedagogically appropriate ways, thereby requiring a close integration of technology, pedagogy, and content as shown in Fig. 2. Recently, a theoretical framework, namely technological pedagogical content knowledge (TPACK), has been developed by Mishra & Koehler (2006) to address challenges of T, P, and C integration. Practicing teachers have been offered professional development (PD) to help them deploy appropriate technologies in the classroom, stay up-to-date with emerging technologies, and assess efficacies of different pedagogical approaches (Loucks-Horsley et al. 2010).

But, due to frequent changes in available tools, challenges might never go away as far as transferring curriculum inventories and PD content to new circumstances. Furthermore, teaching with technology often requires customization and the needed technologies must be both content specific and pedagogically suitable at the same time (Koehler & Mishra 2008). While the latest technologies offer more capacity for applicability, their optimum utilization may necessitate knowledge of tools’ operational underlying principles for easier transfer into new circumstances and better integration (Koehler & Mishra 2008, Niess 2005, Flick & Bell 2000).

It is not very common to come across presentations or papers in teacher education conferences that report use of a pedagogically appropriate technology that is widely applicable to topics in a STEM content area. It is even less uncommon to see one that applies to teaching of topics in multiple content areas. This is what led scientists such as us who heavily used computational modeling and simulation technology (C-MST) in scientific research in the past several decades to cross paths with pedagogy and teacher education experts. We need their help to get more and better students from public schools to enter computational science programs and they need help with interdisciplinary TPACK training of teachers. At the 2014 and 2015 SITE (Society for Information Technology and Teacher Education) conferences, we presented a case study (i.e., CPACK) by demonstrating how we have integrated computational methodology and technology into teacher education. Encouraged by a warm reception and a TPACK paper award (Yaşar et al. 2015) from the SITE education community, we started a fruitful collaboration with other researchers and this has resulted in a better understanding of cognitive foundations of computational modeling and simulations.

There is an important feature of interdisciplinary education that can be best described by Aristotle’s well-known statement, “the whole is more than the sum of its parts,” or the theory of Gestalt psychology, “the whole is other than the sum of its parts,” which means that the whole has a reality of its own, independent of the parts (Koffka 1935). Accordingly, educators have noted an emerging nature of TPACK when technology, pedagogy, and content closely interact (Mishra & Koehler 2006), which is illustrated as the overlap of Venn diagrams in Fig. 2. There is even a stronger case, CPACK, when mathematics, computing, and sciences are integrated through CMST (see Fig. 3). Not only has it given rise to a new content domain of computational science as witnessed by degree programs in the past two decades (Swanson 2002, Little 2003, Yaşar & Landau 2003) but it also led to a particular pedagogy which was not even there among the constitutive domains of computing, mathematics, and sciences to start with (Yaşar & Maliekal 2014a). Below, we explain cognitive foundations of this computational pedagogy.

2.2 Mind as a Computational Device

Modeling and testing has been an important tool for scientific and engineering research for hundreds of years. Scientists often start with a model (e.g., a hypothesis or a concept) deductively based on the current research, facts, and information. They test the model’s predictions against experimental data. If results do not match, they, then break down the model into its parts (sub models) to identify what needs to be tweaked. They retest the revised model through what-if scenarios by changing relevant parameters and characteristics of the sub models. By putting
together new findings and relationships inductively among sub models, the initial model gets revised again. This (deductive/inductive) cycle of modeling, testing, what-if scenarios, synthesis, decision-making, and re-modeling is repeated — similar to the bidirectional distributive/associative structure in Fig. 1 — as resources permit until there is confidence in the revised model’s validity.

In recent years, computers have been very effective in conducting scientific research because they speed up the model building and testing of different scenarios through simulations that provide quick feedback to researchers in order to improve the initial model (NSF Blue Ribbon Report 2006). CMST’s role in scientific and industrial research was proven beyond doubt when computational predictions matched behavior of physical models in high-stake cases (e.g., safety of cars and planes, emissions from engines, and approaching storms). Its use was uniquely justified when a study was impossible to do experimentally because of its size (too big such as the universe or too small such as subatomic systems), environmental conditions (too hot or dangerous) or cost. CMST eventually demonstrated to be generating innovation and insight, just like experimental and theoretical research and this ultimately led to the recognition of computation by the scientific community as a third pillar of doing science besides theory and experiment (PITAC Report 2005).

While such capacity was available only to a small group of scientists in national labs, their demand for computationally competent post-docs and doctoral students led to graduate programs in research universities. A dramatic increase in access to and power of high performance computing and the drop in its cost in the past 20 years helped spread the use of CMST tools into the manufacturing industry. Driven by market needs and trends, rather than empirical research into their effectiveness in education, funding agencies and colleges started investing in new CMST-based BS and MS degree programs across the world (Swanson 2002; SIAM Report 2001, 2007, Yaşar et al. 2000). It was not until friendly versions of such tools were available and considered for use in K-12 settings that a detailed and thorough empirical research was undertaken to measure their effectiveness in education.

If used appropriately, CMST tools can involve students in inquiry-based, authentic science practices that are highlighted in the recent framework for K-12 science education (NRC 2012). A growing body of research (Bell & Smetana 2008; Wieman et al. 2008) identifies computer simulation as an exemplar of inquiry-guided (inductive) learning through students’ active and increasingly independent investigation of questions, problems and issues. Research into the use of computer simulations in science education has been reviewed periodically and quite frequently in educational research in the past decade (2001-2010) and the one by Smetana & Bell (2012). The article by the Rutten et al. (2012) reviewed (quasi) experimental research in the past decade (2001-2010) and the one by Smetana & Bell (2012) reviewed outcomes of 61 empirical studies since 1972. The overall findings support effectiveness of computer simulations. In many ways simulation has been found to be even more effective than traditional instructional practices. In particular, the literature shows that computer simulations can be effective in: 1) developing science content knowledge and process skills, and 2) promoting inquiry-based learning and conceptual change. Effectiveness of CMST in education is also well grounded in contemporary learning theories that recognize the role of experience, abstract thinking, and reflection in constructing knowledge and developing ideas and skills (Hammond 2001; Donovan & Bransford 2005; Illegis 2009; Mooney 2013).

Since CMST is beneficial to both scientists and students in their inquiry and learning, one might wonder in what ways it resonates with the basic functions of the mind. Although the literature suggests linking modeling and simulation to some cognitive functions such as abstract thinking and decomposition skills (Wing 2006), empirical research in cognitive psychology and neuroscience (Brown et al. 2014) encourages us to search further, as there might be a deeper link at more fundamental levels. For example, according to the computational theory of mind (CTOM), the deepest link between electronic and biological (mental) computing devices is a) the common nature of the information that they both process, and b) the way that they process it (i.e., addition & subtraction), regardless of the underlying infrastructure that does the computation (Montague 2006).

Many fields have their hands in the study of how learning takes place in the mind. Cognitive psychologists try to understand how the mind works through empirical research into how people perceive, remember, and think. Developmental and educational psychologists form theories of human development and how they can be used in education. At the same time, neuroscientists use imaging techniques to understand the brain mechanisms that take part in learning. What was started by Alan Turing, the father of computer science, still continues to shed light today on the study of the mind. Basically, Turing’s idea was that if thoughts (i.e., information) can be broken up into simple algorithmic steps, then, machines can add, subtract or rearrange them as our brains do (Montague 2006; pp. 6). Turing also provided an insight that there should be a distinction between the patterns of computations (e.g., computer software and mind) running on a device and the device parts (e.g., computer hardware and brain). His insight keeps fueling the work of computer, computational, and cognitive scientists (Montague 2006; pp. 7). Basically, he laid foundations of a devise that could imitate the mind, thereby giving us a simplified representation (model) of the mind to understand how it would work in different contexts.

While CTOM played a central role within the cognitive sciences during 1960s and 1970s, modern philosophers think that equating mental representations with information processing leaves out the meaning associated with mental events (Montague 2006; pp. 8). We know that CTOM is far from complete, as information processing alone cannot define mental states. But, we also know from scientific research that computational modeling and simulation can generate insight when done in a bi-directional iterative way as shown in Fig. 1. If today’s advanced computer hardware and software have grown to a capacity to generate insight and conceptual change through a structured and cyclic computation with many levels involving various sizes and constructs of information at each level, then we should investigate if the same structure and mechanism support fundamental cognitive processes that may be common to both biological and electronic computation.

In his book, “How We Make Decisions,” the neuroscientist Montague (2006), an ardent supporter of CTOM, describes how the mind attaches value to the computations in order to make meaningful decisions. He argues that the concern for survival pressures us to be efficient in the way we consume our available energy. As an extremely efficient computational device, the brain actually runs on orders of magnitude less electricity than mechanistic computers and mobile devices (p. 26). Furthermore,
he suggests that the concern for efficiency makes us assign “value” to our thoughts, decisions and actions by computing and evaluating different scenarios before we take an action (p. 51). And, that, he thinks is the root of our intelligence and why we have pushed ourselves to be smarter over time.

2.3 Electronic & Biological Computation

Humans have long been curious about how the mind works in ways that are meaningful, plausible, and fruitful for further research possibilities. Studying the mind has been much complicated as it takes place in a delicate, inaccessible, and complicated organ, the brain. However, consideration of the information in terms of simpler and computable pieces by Alan Turing led to an electronic device to imitate the biological brain. After almost a century, the imitation has gotten so complicated, both structurally and functionally, that we may be able to discover how the original (mind) computes by studying how the imitation (computer) does it. Yet, despite similarities of computational processes between electronic and biological computing devices, each uses a different hardware to accomplish what it does. While electronic computers have evolved into distributed structures like the brain’s neural network, there exist many differences. Much of the literature on “computation” today refers to how it is done on electronic devices and it may be time to use the term computation in a device-independent way.

As briefly mentioned in the introduction, the latest neuroscience studies now shed light on how information storage, retrieval (remembering), and processing (thinking) take place by the brain hardware (Brown et al. 2014). While electronic computing machines handle information storage and processing separately through different hardware components, our brains have no separate place for information storage — storing and retrieval are part of information processing (thinking). Both the long-term storage and processing of information involve a synchronized distributed participation of all neurons in related regions of the brain (MacDonald 2008: 97). Programmers of parallel computers know that management and utilization of a distributed hardware necessitates scatter and gather type communication functionalities in software. That is similar to what is going on in the brain circuitry. When new information arrives, it lights up all related cues, neurons and pathways in a distributive process that is similar to the top-down action in Fig. 1, where new concept is broken up into related pieces. With the same token, retrieving a memory is a reassembly of its original pattern of neurons and pathways in an associative process that is similar to the bottom-up action in Fig. 1. Retrieval is often regarded as an act of creative re-imagination and what is retrieved is probably not the original pattern but one with some holes or extra bits (Brown et al. 2014: 75, MacDonald 2008: 101). Neuroscientists argue now that there is no distinction between the act of remembering and thinking (MacDonald 2008: 97).

The distributive and associative way of information processing by the brain circuitry is consistent with the dual deductive and inductive process of computational modeling and simulation that we discussed in earlier sections. While the brain’s neural circuitry offers a chance for full utilization, the efficiency, intactness, and effort-fullness with which it is used depends on each individual. A scientist is a good example of a person who exercises this bi-directional thinking methodology in a complete cycle. Since the latest learning theories recommend that student learn science the way a scientist does his inquiries, these thinking skills should then be taught to young learners. They are actually part of the electronic computational thinking (CT) skill set as described by Jeannette Wing (2006). Some of the currently described CT skills may be grounded in cognitive processes that we have discussed here. For example, the decomposition skills of CT roughly correspond to the distributive, deductive, and top-down cognitive process of information we have described here. And, the abstraction skills roughly correspond to what we have described as associative, inductive, and bottom-up cognitive process of information.

Abstraction is an inductive process, whereby details are filtered out and focus is placed on more general patterns, thereby allowing one to assign priority and importance to the newly acquired information. Researchers find it amazing that we make strong generalizations from sparse, noisy, and ambiguous data (Tenenbaum et al. 2011). Abstraction helps our cognition, especially at its developmental stages, by simplifying, categorizing, and registering key information and knowledge for quicker retrieval and processing (Bransford et al. 2000). Perhaps, we developed abstract thinking skills as a result of a survival concern for having limited resources (i.e., time, memory, attention). Our tendency to summarize and generalize information — before we permanently store it — might be a strategy to overcome limited storage capacity. Such tendency can shield us from details that have no practical value for survival. Another evolutionary idea is that the brain’s tendency to process information in a dual fashion might be because it has sought a way to adjust to dual behavior of matter and the incoming information that reflects matter’s dual behavior. Whatever the origins are, findings in neuroscience indicate that it is not just the limited capacity of our brain or our survival instinct but also the distributed structure of the brain hardware that drives a bi-directional (distributive and associative) flow of information, which results in tendencies that benefit us.

The growth of our brain hardware and software is a bit complex and many things can go wrong during a lifespan. Normally, at birth, the circuitry at the inner part of the brain is up and running to manage vital and involuntary functions (e.g., breathing, heartbeat, and some degree of sound and visual tracking), but the outer part (cerebral cortex) takes some time to be ready for voluntary actions (e.g., conscious thought, information storage and processing) (Restak 2001). Actually, the majority of neurons that a human is born with are contained within this thin cortex that separates humans from other animals. While only a few neurons develop during adulthood, we can take comfort that mental growth is not solely based on the number of neurons in the brain, but rather the increasing complexity of the connections between them. Other factors that affect mental growth include the functionality that each neuron or groups of neurons assume, the size they grow into, and the placement in different parts of the brain that they migrate towards. Even more important is the number of inter-neuronal connections, which are estimated to be near 100 trillion. New neural connections are being made all the time as we learn new things. In fact, these connections constitute the definition of learning, and the existing connections are strengthened, weakened, or even eliminated if not revisited often enough. Genetics plays only a partial role determining the growth of the brain, as there are not enough genes on the human chromosome to code for the placement of billions of neurons and trillions of connections (Restak 2001). This luckily leaves plenty of room for the brain (and the mind) to continue growing as a result of one’s free will, experience, and environment.

So, the good news is both deductive (e.g., decomposition) and inductive (e.g., abstraction) thinking skills can be improved
There is no doubt that factual details in science and mathematics coursework are often overwhelming, causing high degrees of frustration for some students. Such individuals perceive science and mathematics topics to be more complex than they are and abandon their pursuit altogether. However, learning can be a joyful activity, if one is predisposed to delayed gratification, which is seldom the case with middle and high schoolers. Hence teachers everywhere face challenges that are daunting. Perhaps, there are two ways to overcome this. One of them requires a cultural change to teach new generations how to become effortful learners and predispose them to delayed gratification. This would take a whole village to do. And, it might take a lot longer than we have come to know Vygotsky’s theory, which says pushing a learner to reach his potential is a lot more important than giving him freedom to choose between effort and withdrawal. This would be like swimming against the flow in today’s educational system and cultural setting. The other option requires a pedagogical practice to employ a general simplistic framework from which instructors can introduce a topic and then move deeper with more content only after students gain a level of interest to help them endure the hardships. As explained in the next section, educators have often opted for this latter deductive approach.

Teacher organizations and national standards (Bell et al. 2008) have suggested ways to create “antidotes” from the very thing (technology) that is known to have caused distraction and a tendency for an easy living. At the same time, the latest learning theories suggest that students should learn science the way scientists do their work (Bransford et al. 2000). For example, the framework for next generation science standards (NRC 2012) suggests that students learn better if they are engaged in activities closely resembling the way scientists think and work. If we combine these suggestions — that is, using technology with the way scientists conduct their work — we would recall from Section 2.2 that scientists today heavily use CMST to do their work. So, the antidote can be computational modeling and simulation but it has some strings attached to it according to a national report (NSF Report 2008). Young learners cannot use the same CMST tools that the scientists use, as they might need prerequisite knowledge that they surely will not have. The report states that at early stages computational modeling approach should involve easy experimentation (learners must be able to quickly set up and run a model using an intuitive user interface, with no knowledge of programming or system commands) and high interactivity (models need to evolve quickly and include smooth visualizations for providing interactions and feedback to users).

Modeling is a simplification of reality — it eliminates the details and draws attention to what is being studied. It enables the learner to grasp important facts surrounding a topic before revealing the underlying details. Tools, such as those in Table 1, now make it possible for instructors to offer easy experimentation in the classroom without having to expose students to STEM principles. For example, as described in later sections, Interactive Physics (IP) and AgentSheets (AS) can be used to create many fun things that could engage students into science experimentation, either by modifying an existing model or creating one from scratch.

### 2.4 Learning Processes Supported by CMST

The issue of why STEM subjects may not be as engaging as others is complex. According to a study in 20 developed countries (Sjøberg & Schreiner 2005, Osborne & Dillon 2008), student attitudes towards science become increasingly negative as a country advances economically. The study suggests this phenomenon to be deeply cultural. Born in the early-to-mid 20th century as a reaction to the rigid and formal style of discipline-based education, today’s progressive education system in the U.S. has come to engage students by making learning fun and exciting (Mooney 2013). There is nothing wrong with that. However, learning some subjects, such as science and mathematics, can be overwhelming because it involves factual details and requires application, discipline and delayed gratification — values the contemporary culture does not seem to encourage. Effortful learning is the key as we discussed earlier, according to the latest research in cognitive sciences and neuroscience. While the need for guiding young minds into the process of effortful learning had already been theorized by Vygotsky around the time of progressive education movement in America, the theory did not find its way across the Atlantic until two decades ago (Mooney 2013; Hammond et al. 2001).

| Excel Spreadsheets: | conduct modeling and simulations using a simple algebraic (new = old + change) for rate of change. |
| Texas Instruments (TI) Tools: | advanced graphing tools to conduct algebra, functions, and rates of change |
Simulation adds another level of benefit on top of easy modeling by providing a dynamic medium for the learner to conduct scientific experiments in a friendly, playful, predictive, eventful, and interactive way to test hypothetical scenarios. For example, in a harmonic motion of an object attached to a spring (Fig. 4), IP can provide control buttons to change physical parameters such as string constant, mass of the swinging object and its initial velocity, intensity of gravitational acceleration, among others. It also gives the user the ability to change some operational parameters, such as the run-time and accuracy desired from the simulation. Furthermore, it allows the learner to go into the initial model’s details and break it into its constitutive parts in order to run various what-if scenarios. Based on these scenarios and their outcomes, the learner can go back to the design phase and change the model (spring and box) to his desire. This dynamics of making decisions that lead to modifications to the initial model based on what-if scenarios is an inductive process because it lets the learner to put pieces of the puzzle to come up with a revised model. When used together, then, modeling and simulations affords the learner the opportunity to cycle iteratively back and forth between the inductive and deductive approaches to learning (Yaşar & Maliekal 2014). This resonates with how the mind itself works because it, too, uses a similar dual methodology (distributive and associative) in its information storage and processing as we explained before.

![Figure 4](image-url)  
Figure 4. A typical user-created simulation in Interactive Physics: harmonic motion of a box attached to a spring on a flat surface.

### 2.5 Deductive & Inductive Approach to Instruction

There are many advantages of deductive and inductive approach in teaching and learning. The deductive approach to instruction entails the teacher introducing a new concept or theory to students by explaining it first, then showing an application or two of the theory or concept, and wrapping up the instruction by affording students an opportunity to apply the theory or concept by completing homework problems (Prince & Felder 2006). This has been and continues to be the traditional approach to science instruction, and it often leads to apathy and eventual attrition of students. The inductive approach to instruction, by contrast, first presents students with a problem, a case, or data from an experiment. Students are then guided to explore underlying facts, issues and the like. As the culminating step, students are led to acquire on their own an understanding of the underlying concept or organizing principle (Prince & Felder 2007). Inquiry-guided learning, problem-based learning, and project-based learning are all among forms of inductive instruction. While empirical evidence suggests that the inductive approach to instruction is superior and that it fosters greater intellectual growth (Bransford et al. 2000, Donovan & Bransford 2005), prudent educators should take advantage of different approaches of teaching.

Modeling- and simulation-based computational pedagogy carries many characteristics of the constructivist approach (Grabinger & Dunlap 1995), including inquiry-based, generative, cooperative, and interactive learning as well as project and team based instruction. Creating a model through step-wise process and running it at each stage of the development have the added advantage that learners get immediate feedback about their work. It may be used in situations when learning about the underlying theories and mathematical concepts that are important. Through this process, learners can be led to develop an understanding of scientific reductionism that studying a system or solving a complex problem requires breaking the system into its components or the complex problem into smaller chunks (i.e., decomposition). Using models and simulations, learners become actively engaged in “doing,” rather than passively “receiving” knowledge. In so doing, the learner becomes the center of the learning process, allowing self-interpretation of the problem and revise it if necessary, mediated by own biases, beliefs, preconceptions, prior knowledge and observations. Once learners successfully infer an organizing principle or theory, they can embark on the next logical and necessary step; one that involves predicting the consequences of the organizing principle or theory that learner just inferred and ascertaining whether the organizing principle or theory is viable, given the consequences. Anyone who learns in this fashion would, in fact, be practicing the craft of scientists (Wieman et al. 2008).

Because simulation modules of differing complexity and flexibility have already been developed and made public, it is now possible to lead learners to perform a series of simulations to explore a scientific process in a manner that is similar to how scientists conduct controlled experiments, by holding all except one variable invariant. A teaching and learning method reliant on CMST is being welcomed by today’s traditional college and school students, as they are digital natives, attracted to and captivated by all things digital! Even non-science students, with no prior knowledge of physics, who used CMST tools and web-based simulations, have shown the ability to provide good explanations of scientific phenomena much more quickly (within hours) than physics majors after a year of physics (Wieman et al. 2008). So, having believed in the promise of dual pedagogical aspects of CMST, we ran a professional development program for in-service and pre-service teachers, hoping that it would engage teachers in their profession and improve both the teaching and learning in their classroom. The next section will detail implementation of our decade-long program along with data collected and analyzed by independent evaluators.

### 3. IMPLEMENTATION & KEY FINDINGS

While the results of our CPACK professional development program have already been documented in earlier publications, such as Yaşar et al. (2014), their importance for and relevance to the aforementioned theoretical frameworks have gradually come to our attention in recent years as a result of our work in pedagogy and cognitive sciences. In this section, we briefly review findings on teaching and learning that are relevant to our discussion.
While the main activity of our study has been teachers’ computational pedagogical content knowledge professional development, the ultimate desired outcome was better student engagement and learning as well as teacher engagement/retention and teaching. A mixed-methods approach (Creswell 2012) was used to collect and analyze qualitative data (interviews, activity logs, observations, pre- and post-activity surveys, and artifacts) as well as quantitative data (student grades and report cards, test scores, and standardized exams by the NY State) for the purpose of formative and summative assessment.

Integration of modeling and simulation tools, such as those in Table 1, into secondary school teaching was initially done in three steps by incrementally adding a new domain of knowledge each year for the first three years. As shown in Table 2, the first step of the multi-tier incentive-based PD included technological knowledge (TK) training, the second step included technological content knowledge (TCK) training, and the final step included teaching of content through computational and pedagogical tools. Here, technology knowledge (TK) means knowledge of technology tools and their use. Technological content knowledge (TCK) means integrating knowledge of technology and STEM (physics, chemistry, biology, math, etc.) for the purpose of teaching its content. Technological pedagogical content knowledge (TPCK) means applying pedagogical technologies to the teaching of STEM content.

Table 2. Profiles of teachers from Urban (U) and Suburban (SU) School Districts at the CPACK summer training (2003-2007).

<table>
<thead>
<tr>
<th>Training</th>
<th>TK</th>
<th>TCK</th>
<th>TPCK</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>School</td>
<td>U</td>
<td>SU</td>
<td>U</td>
<td>SU</td>
</tr>
<tr>
<td>Math</td>
<td>96</td>
<td>14</td>
<td>42</td>
<td>2</td>
</tr>
<tr>
<td>Science</td>
<td>38</td>
<td>15</td>
<td>17</td>
<td>9</td>
</tr>
<tr>
<td>Tech</td>
<td>7</td>
<td>3</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>Special Ed</td>
<td>14</td>
<td>1</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>TOTAL</td>
<td>155</td>
<td>33</td>
<td>66</td>
<td>12</td>
</tr>
</tbody>
</table>

Supported by the National Science Foundation through various grants, we formed a CMST Institute in 2002 and have since been offering CPACK PD to in-service and pre-service secondary school teachers. The professional development program has both summer and academic-year components. While we constantly explore new tools, we continue to use those in Table 1 because of the large database of artifacts and lesson plans we have developed using them over the past decade. Table 2 shows the number of in-service teachers who benefited from the summer institute component offered through NSF support in partnership with local school districts (Rochester City School District (RCSD) and Brighton Central School District (BCSD)) and several national organizations (Shodor Foundation, Krell Institute, and Texas Instruments). Almost half of teachers who attended TK training returned for additional TCK training, and half of those returned for TPCK training. This is typical of an incentive-based PD (Loucks-Hersley et al. 2010). Teachers have multiple summer engagements and some teach in district summer schools. So, the dates and time impact attendance. For those who could not attend due to such circumstances, we offered similar short courses during the school year. The partnering districts also offered a condensed version of the training to additional 160 teachers through turnkey training and PD days. For the purpose of gathering data for research and evaluation, we only worked with teachers who attended the summer institute as part of commitment to the study.

The initiative displayed elements of a scalable innovation (Dede et al. 2005), especially in mathematics. There was a cultural change in all 15 secondary schools at the urban RCSD and the suburban BCSD. They were fully engaged all the way from superintendents and principals down to teachers and students. Improved teacher retention and student achievement reported by partnering districts drew national attention to this initiative, including testimony by the author, Jeff Mikols (a RCSD math teacher who is now a district curriculum director), and Ed Chi (a BCSD science teacher who has left the district) before the U.S. Congress (House Hearing 2003).

In a 2010 survey of 40 TCK and TPCK teachers, 94% agreed that the training made them more effective in the classroom; 87% agreed that it strengthened their pedagogical skills; 73% agreed that it strengthened their pedagogical content knowledge; 100% agreed that training strengthened their skills related to modeling and simulation; 86% reported that they continue to use the hardware, software and other materials made available through the project in their classrooms; and 80% believed that their participation served to build leadership skills. Seven years after the start of the initiative, 73% of participating teachers at RCSD were still teaching while 10% had moved to lead positions (Yaşar et al. 2014). According to the National Center for Education Statistics (NCES 2014), about 16% of STEM teachers either move to another school or leave the profession every year. The national average is that nearly half of all new STEM teachers leave the job within five years (Graziano 2005). Although we do not have the 2002 baseline data from participating districts to compare with, urban schools such as RCSD generally perform much worse than the national average. RCSD district officials reported throughout the initiative (Crowley 2007) that it not only helped retain veteran teachers but it also drew more and better teachers to an urban school district, which usually has a hard time recruiting teachers because of the well-known urban problems (Margolis et al. 2008).

Table 3: Frequency of technology tools used by trained teachers.

<table>
<thead>
<tr>
<th>Subject /Grade</th>
<th>Daily</th>
<th>Weekly</th>
<th>Bi-weekly</th>
<th>Special Projects</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math Grades 7-8</td>
<td>Laptop, smartboard</td>
<td>Power Point, PI tools, GSP, Excel, Flash</td>
<td>AgentSheets</td>
<td>Interactive Physics (IP), Stella, Java, GIS/GPS</td>
</tr>
<tr>
<td>Math Grades 9-12</td>
<td>Laptop, smartboard, TI tools</td>
<td>Power Point, PI, Excel, Flash</td>
<td>AgentSheets, Excel, TI</td>
<td>IP, Stella, Java, GIS/IPS</td>
</tr>
<tr>
<td>Science Grades 7-8</td>
<td>Laptop, smartboard, Power Point</td>
<td>AgentSheets, Excel, PI</td>
<td>TI, GIS/GPS, Flash, Java</td>
<td>Stella, GSP, Interactive Physics (IP)</td>
</tr>
<tr>
<td>Science Grades 9-12</td>
<td>Laptop, smartboard</td>
<td>Flash, Excel, Power Point</td>
<td>Interactive Physics (IP), Java, GPS</td>
<td>Stella, AgentSheet, GIS, PhET</td>
</tr>
</tbody>
</table>

All of the trained secondary school (grades 7-12) teachers reported that on a daily base they used laptops for presentations, graphing calculators for math instruction, and electronic smart boards for interactive lessons (see Table 3). Positive experience with C-MST tools is believed to have initiated use of additional tools such as GIS/GPS, Java, Flash, and PhET (Wieman et al. 2008). Annual surveys of teachers showed that usage of the tools in the classroom was directly linked to the amount of training they had received. In post-training journals, while only 60% of the teachers reported occasional use of modeling tools in their...
classrooms after the initial TK training, 78% reported that they used them regularly after the TPCK training.

Table 4: Percent of teachers using modeling in class

<table>
<thead>
<tr>
<th>Grade Level</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Regularly</td>
</tr>
<tr>
<td>7-8 Math</td>
<td>46%</td>
</tr>
<tr>
<td>9-12 Math</td>
<td>60%</td>
</tr>
<tr>
<td>7-8 Science</td>
<td>25%</td>
</tr>
<tr>
<td>9-12 Science</td>
<td>54%</td>
</tr>
</tbody>
</table>

In a 2007 survey by 65 active teachers who had received at least two years of training, many reported a significant use of modeling tools for both classroom instruction and special projects (see Table 4). It appears that the higher the grade level, the more regularly these tools are used in the classroom. Less frequent use of tools in RCSD middle school science classes was a concern, which resulted from access and scheduling problems but it got better over time as the concern was conveyed to the district administration. At BCSD, access to computing resources was not an issue. For example, participating teachers ended up fully integrating Interactive Physics into their high school physics labs.

Figures 5 through 8 show some of the survey results in graphical format regarding student engagement and learning as a result of CMST-enhanced teaching. More than 92% of surveyed teachers agreed that computational inquiry made math and science concepts significantly more comprehensible to students (Fig. 5).

Figure 5: Improved comprehension of STEM concepts.

Figure 6: Deeper understanding of STEM concepts.

100% of technology, 72% of math, and 31% of science teachers reported observed improvement in students’ problem solving skills. Student reaction to modeling (versus traditional techniques) was found to be 97% favorable in math and 77% in science classes. While science classes utilized technology less due to limited access and lack of science-related modeling examples, in instances where it was utilized, a deeper understanding of science topics was achieved, compared to math topics (83% vs. 76%, see Fig. 6). As seen in Fig. 7, students in higher-grade levels found computational modeling more engaging in both math classes (grades 7-8: 77% vs. grades 9-12: 90%) and science classes (grades 7-8: 75% vs. grades 9-12: 85%). Modeling was even found helpful to non-traditional (special education) learners (Fig. 8); again the higher the grade level the higher the engagement: math classes (grades 7-8: 76% vs. grades 9-12: 100%) and science classes (grades 7-8: 75% vs. grades 9-12: 85%).

Figure 7: Student engagement per grade level and subject.

Figure 8: Impact on non-traditional learners.

Qualitative data from journal entries, activity logs, and teacher interviews pointed out to an emerging pattern regarding gender response to CMST-based teaching. Two independent coders read the 2010 teacher survey data and coded the text segments to arrive at descriptions and common themes. An inductive process (Creswell 2012) was used to group these codes in order to form even broader themes. Based on detailed accounts of 26 teachers (out of 40), the evaluators arrived at the following broad theme:

While male students showed more interest in playing with technology and plowing through the details with less regard to the.big picture, female students initially seemed reluctant and timid but excelled when details (curriculum) were put into context of real-world problems and projects.
This is consistent with national findings by our collaborators such as Repenning (2012). It is also consistent with our own data when triangulated against student scores and graduation rates. For example, while cohorts of 8th grader male and female students from both districts had a gap in their average math performance at the beginning of the initiative, not only were the gaps closed but also reversed four years later (12th grade) as shown in Table 5. At RCSD, while both male and female students did much better than four years earlier, the graduation rate of the same cohorts still reflected a gender-based trend in performance growth, favoring female students. To examine whether the difference is statistically significant, we calculated the z-scores assuming a normal distribution approximation (Brase & Brase 2012). The sample sizes for male and female students were roughly the same at both districts, with about 1200 at RCSD and 150 at BCSD. The column p indicates the confidence level that the difference between males and females may be due to a nonrandom effect. Normally, any confidence level below 90% is less than significant. Here, with more than 90% confidence level female cohorts outperformed male cohorts in both math performance and graduation rates.

Table 5: Gender-based performance history at RCSD & BCSD.

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Gender</td>
<td>Gender</td>
</tr>
<tr>
<td></td>
<td>M</td>
</tr>
<tr>
<td>RCS D</td>
<td>Math</td>
</tr>
<tr>
<td>Graduation Rate</td>
<td>34%</td>
</tr>
<tr>
<td>BCS D</td>
<td>Math</td>
</tr>
<tr>
<td>Graduation Rate</td>
<td>85%</td>
</tr>
</tbody>
</table>

To further triangulate self-reporting data by teachers, annual student achievement data were analyzed in the partnering school districts via report cards and standardized test scores. While we cannot fully isolate the impact of teacher training from other contributing factors, an upward district-wide trend was noted in both urban and suburban districts during the initiative. The percentage of students receiving a Regents diploma increased significantly from the baseline (RCSD: 21% → 59%, BCSD: 84% → 95%). The initiative exposed students from the urban district to college experiences and opportunities, and this may have led to an increased interest (78% → 83%) in both 2-year and 4-year college enrollments over the period examined. Furthermore, the passing rate (>65/100) in NY State Grade-8 Math exam increased in Rochester City SD from 10% to 33%, while the passing rate in NY Regents Math-A exam (Grade 11-12) also increased from 13% to 67%. Passing rate in sciences also increased in areas such as Physics (3% → 22%) and Chemistry (9% → 27%). At BCSD, passing rates improved in mathematics (Math-A: 51% → 99%) and sciences (Physics: 52% → 78%). The number of students taking General Physics at Brighton increased from 50% to ~100% and the number of students taking AP Physics also doubled. Student passing rates at both districts seemed to reflect relative participation of district’s math and science teachers in the initiative. All of the improvements have been found to be statically significant for typical sample sizes from each district.

The main goal of the sponsoring No Child Left Behind program was to train as many teachers as possible to potentially create a district wide impact on student achievement scores. As a result we trained twice as many as we had committed to (see Table 2).

While the goals of the sponsoring agency were met, as witnessed by gains in the standardized test scores reported by partnering districts, no comprehensive research was done by the project to more closely link the gains in student achievement scores to the teaching and learning resulted from the initiative. By the time the goals of sponsoring NSF program shifted from ‘leaving no child behind’ outreach to ‘researching the interventions’ we had almost run out of control groups in partnering school districts’ math classrooms. The initiative invited science teachers but limited access to computer labs, skepticism about use of technology, and inadequate number of readymade curricular modules discouraged many to invest in trainings that lacked significant science content and representative lesson plans. By the end of the project while almost all secondary math teachers in RCSD and BCSD received training and yearlong PD, only 20% of science teachers took part.

In final years of the study, when focus shifted towards researching the intervention, a few treatment-control comparisons were conducted. A pair of CMST and non-CMST high school teachers from the same school taught properties of quadrilaterals in a mathematics class. The CMST teacher used GSP in a class of 24 pupils while the non-CMST teacher used conventional methods in a class of 14 pupils. Both teachers conducted the same unit test. Even though the CMST teacher taught a more crowded class, his classroom average was 82.5 versus 49.5 for the other class. The second study involved a math triathlon similar to Regents Math A and B tests involving use of TI graphing calculators. Scored by external judges, including teachers and college faculty, this study revealed that students taught by CMST teachers outperformed other students in all categories: Math-A: 60.26 vs. 49.54; Math-B: 71.9 vs. 55.6; and 7-8 Grade Math: 64.0 vs. 58.6.

Over the past decade, institute staff and participants created a large database of more than 300 CMST curriculum modules and lesson plans. Curriculum modules and lesson plans from the database have been downloaded by people around the world at a rate of 50-80 per day, totaling almost 100,000 since the database was launched. The database has also provided content for two local pre-service methods courses (NAS 401/501 C-MST Tools and NAS 402/502 Computational Pedagogy) in the college’s teacher education program. Table 6 shows pre-service enrollments in these credit-bearing NAS courses. Additionally, the database supported turnkey training offered by partnering districts during professional development days, serving 160 in-service teachers.

The CMST database (www.brockport.edu/cmst) continues to support three general education courses reported earlier in this journal (Yasar 2013). They have since served 500 more STEM undergraduates. The two NAS methods and 3 general education courses have become part of the NSF Robert Noyce Scholarship program since 2012, serving a new cadre of 50 computationally competent STEM teachers, some of whom have already started teaching in high needs school districts both locally and nationally.

Table 6. Number of pre-service teachers trained.

<table>
<thead>
<tr>
<th>Courses</th>
<th>2003-07</th>
<th>2008-12</th>
<th>2013-15</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-MST Tools &amp; Pedagogy</td>
<td>113</td>
<td>107</td>
<td>105</td>
<td>325</td>
</tr>
</tbody>
</table>

In Rochester City and Brighton Central secondary school classrooms taught by CMST teachers, students were all given a chance to experience the deductive and inductive learning processes. As mentioned earlier, 97% of mathematics and 92% of sciences classes using the CMST approach agreed that it made subject-related concepts more comprehensible. Furthermore, 83%
of science classes and 76% of math classes found that it led to
even a deeper understanding of STEM concepts. While modeling
is a common practice in mathematics and science classes, science
classes often go beyond modeling to utilize simulations in order to
investigate time-dependent dynamics of scientific phenomena.
When used together, modeling and simulation affords the learner
a constructivist opportunity (Grabinger & Dunlap 1995) to cycle
iteratively back and forth between the inductive and deductive
approaches to learning (Yaşar & Maliekal 2014). Teaching
mathematical and computing concepts contextually has been
recommended for quite some time by national learning standards
(NGSS, Computing Curriculum 2005) but we now additionally
know from cognitive sciences that retrieval practices attempted at
various contexts is good — because every time the recall is
attempted in a different context, it establishes more links that will
help the remembering and learning (Brown et al. 2014).

Benefits of constructivist and contextual learning was observed in
an annual after-school CMST challenge competition, which
allowed students more time and freedom than a regular classroom
setting to apply, test, and revise the constructed computational
models. Participating students had a full semester to develop a
team project. Scoring rubric included problem statement,
application of the model to a problem of interest, data analysis,
teamwork, originality, electronic demonstration, and presentation
of the results before a panel. Extra points were given for use of
multiple CMST tools, demonstrated understanding of
computational, mathematical and scientific content, and level of
challenge, knowledge and skills demonstrated beyond team’s
grade level. As expected, the incentives helped push students to
go beyond initial job of model construction, playful
experimentation, and introductory exposure to STEM concepts. A
project-based experience reported in Yaşar (2005) by a group of 9th
grade high school students from Brighton High School (NY), who used the Interactive Physics and Geometer’s
Sketch Pad to prove Kepler’s Laws in an afterschool program
(annual CMST Challenge), is a testimony of how students gained
a deeper understanding of computational and scientific content of
the planetary motion. Following is a sentiment by these high
school students after their CMST experience to prove Kepler’s
laws:

“We had not taken any physics courses and we were not fully
knowledgeable about laws of universe that govern planetary
motion. That is not different from the situation of Kepler; as
no one quite knew how gravitational forces worked until
Newton came. Kepler had access to data compiled by Tycho
Brahe and he looked for patterns. We had access to modern
tools and we looked for miracles! We learned how to transfer
visuals images and data from Interactive Physics to
Geometer’s Sketchpad to measure angles, distance, and areas
of triangles needed for the proofs... While it was initially
frustrating to learn new tools, realizing what Kepler would
have done if he had such tools; we quickly learned to
appreciate the opportunity in our hands. In the end, we did
not make a discovery in physics, but we certainly discovered
that physics was not a threatening or boring subject. We also
discovered the role of mathematics in physics. The foreboding
nature of complicated physics was abolished and we all
looked forward to taking physics classes.”

The authors followed progression of these students as a case
study. In their project the following year, these 10th graders
inquired further about fundamental STEM principles of their
projects and operational principles of the tools they used for
modeling and simulations. Using Excel to compute a simple
algebraic form of rate of change equation, \( \text{new} = \text{old} + \text{change} \),
that they had learned in the mathematics class that year, they
attempted to replicate the Interactive Physics results found earlier
for the harmonic and planetary motion. For the harmonic motion
in Fig. 4, this involved computing algebraic formulas for the
position \( (x_{\text{new}} = x_{\text{old}} + dx) \) and velocity \( (v_{\text{new}} = v_{\text{old}} + dv) \) of the
spring-driven object at times \( (t_{\text{new}} = t_{\text{old}} + dt) \) separated by interval
dt. While time \( (t) \) was an independent variable, and change in \( x \)
was dependent on the velocity as \( dx = v \cdot dt \), and the change in \( v \)
was dependent on the acceleration as \( dv = a \cdot dt \). Where acceleration \( (a) \) is
Force/mass. The force applied by a spring unto
an attached box is \( F = -k \cdot x \), where \( k \) is the stiffness coefficient of
the spring and \( x \) is the displacement of the box from the
equilibrium position \( (x=0) \). The details of their self-constructed
simulations is given in Yaşar et al. (2006), yet the brief statement
below summarizes the progress they had made — they were no
longer threatened or frustrated by learning of science.

“Through Excel, we were able to use a simple algebraic
equation \( (\text{new} = \text{old} + \text{change}) \) to manually construct our
own simulations as an alternative way and compared them to
those done earlier by the Interactive Physics. To compute the
‘change’ all we needed was some basic knowledge of the
force that governed the system, whether it was the harmonic
or the planetary motion.”

The progression by these students show that the learner can start
either with a readymade model, or construct one using a pull-
down menu, that represents the scientific phenomenon under
study and conduct fun experiments without having to know the
details of the model and the laws that govern its motion. If it stops
there, then we can say that the top-down deductive approach has
gained students in STEM activities. But, if the learner is tempted
to continue and inquire about the initial model’s constitutive parts
and forces that act on them, then he can run simulations by
changing characteristics of the parts and forces to inductively
construct a new model and physical setting that better represent
the reality. This cycle can be repeated until the desired knowledge
or outcome is reached. This way of learning, through inquiry and
experience, is nothing but how scientists do their work (Bransford
et al. 2000, Donovan & Bransford 2005). Such an iterative and
stepwise progression in constructive learning is also consistent
with several pedagogical frameworks, including scaffolding, zone
of proximal development (ZPD) that we discussed earlier by
Vygotsky, and the Optimal Flow (Csikszentmihalyi 1990) shown in
Fig. 9, which suggests the importance of balancing challenges
and abilities using pedagogical stepping-stones in order to attain
optimal flow for a learner.

![Figure 9: Illustration of Optimal Flow as a path of learning.](image-url)
4. CONCLUSION
Cognitive psychology research has shown that interleaved retrieval practice has great advantages for gaining deep and lasting knowledge. Interdisciplinary education is a form of this practice at course and curriculum levels but it takes effort and time, thereby slowing down the learning process. In college, it delays graduation and in K-12 it slows down the pace of teaching. Technology can speed it up but this throws another ranch into the delays graduation and in K-12 it slows down the pace of teaching. We have run a decade-long experiment to study the task explained works by adding another domain of knowledge. So, the question becomes of finding a technology that will facilitate mixing of multiple views around a topic in a pedagogically way. This, we claim, calls for the use of computational modeling and simulation technology because it naturally adds a deductive and inductive pedagogy to teaching of STEM content. The final question, then, becomes, “OK, we got this wonderful thing, how do we go about institutionalizing it?” And, this is where the need for teacher training becomes the central task, as they are the agents of change for any reform in the schools (Bybee & Loucks-Hersley 2000, Loucks-Hersley et al. 2010).

We have run a decade-long experiment to study the task explained above, using CMST tools within an interdisciplinary CPACK framework for teacher professional development. Triangulated data from multiple sources indicated that the use of CMST tools and pedagogy not only supported basic interleaved retrieval practices but it enriched such practices by putting the learner on the driver seat through an iterative cycle of constructivism, interactivity and immediate assessment. Not only did this cyclic process helped students: a) engage in a topic through a general simplistic introduction and b) move deeper deductively into more content as they gained more skills, but it also enabled them to construct significant knowledge through easy experimentation to inductively draw conclusions about the topic they started with. Computational modeling and simulation involves all of these as demonstrated in our initiative in public schools. The deductive aspect of modeling helped teachers present science concepts to learners by simplification of reality, which was instrumental to draw young minds into science learning. High levels of student engagement reported by our participating teachers strongly support the effectiveness of computational modeling as a deductive pedagogical tool. The CMST tools did exactly as expected by shielding students from having to know detailed content knowledge of mathematics (e.g., differential equations), computing (e.g., algorithmic and programming) and science (e.g., physics) to conduct experiments of linear, harmonic, and planetary motion using IP. The inductive process resulting from experimentation through simulations helped learners to rediscover principles of computing and sciences, therefore leading to deeper content learning. Since it is the inductive reasoning that help us come up with general patterns and simplifications from paralyzing details, one cannot have a chance to utilize a deductive approach if there had not been an inductive counteract to simplify concepts for later use. So, we do not have an option of choosing one over the other in education; we need to use both, as they complete — not compete with — each other. Improved student achievement scores in both local and statewide exams at partnering school districts point out to a lasting impact of the dual nature of computational pedagogy.

Our initial focus on pedagogical aspects of CMST was to develop a tool-independent CPACK training for our teacher education program in order to maximize transfer of curriculum inventories to new conditions when newer technologies become available. However, we stumbled upon much more. Information revolution has taken electronic computing devices to every corner of the globe but very few would be familiar with and relate to computational modeling and simulation. In fact, even some researchers and educators might consider CMST as an ad hoc technology. Computing is not usually considered as a branch of science (Denning 2009) because it deals with artificial phenomena, not natural phenomena. However, as artificial and imitational as electronic computation is, it might actually help us discover how the biological computation generates complex mental states. We think it is going to do more than that, as understanding how pervasive the computational behavior is might change the way we relate to ourselves and everything else in the universe.

Computational theory of mind considers electronic and biological computing devices to compute the same way at the fundamental level, but much is needed to reduce our complex mental states to mere computational processing of information. Regardless of what high level processes a computing device is performing, we think that the way computing is done at the most fundamental level will carry itself all the way to the top level. Computational modeling and simulation is a high level electronic process whose dual characteristic does reflect the two fundamental modes of computing (i.e., addition and subtraction). Deductive and inductive thinking, on the other hand, are also two high-level cognitive processes that similarly reflect the same modes of computation. So, one can suggest that it is the computable nature of information that leads to commonality of electronic and cognitive outcomes of computing regardless of the underlying structure. A million-dollar question would then be ‘what is the source of information’s computable (associative and distributive) behavior?’ Is it merely reflecting how the matter itself behaves?

Computability actually appears to be a universal characteristic of both granular matter and quantifiable information. Anything quantifiable has three distinguishable outcomes: quantity, sequence, and pattern. If quantifiable stuff — be it matter or information — can form various patterns to make up atomic and cellular structures as well as instructions and thoughts, then everything we see out there is computable (Montague 2006; pp. 14). If so, then perhaps we can start examining a computational theory of everything (Yaşar 2016) that would mean everything in the universe behaves computationally by either uniting with (addition) or departing from (subtraction) other things to form a new sum as, again, depicted in Fig. 1. Our current and future studies will continue along these lines. Any traction that it might gain will be a tribute to Turing.

5. ACKNOWLEDGEMENT
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6. REFERENCES
